

1.

$$\vec{a} = (3, 4, 2)$$

$$\vec{b} = (1, 1, 1)$$

$$\vec{c} = (-2, 5, 1)$$

Solutions

(scaler)

$$a) (\vec{b} - \vec{a}) = (-2, -3, -1)$$

$$b) (\vec{a} + \vec{c}) = (1, 9, 3)$$

$$c) (\vec{a} + \vec{b} + \vec{c}) = (2, 10, 4)$$

$$d) (2\vec{a} - 3\vec{b} + \frac{1}{2}\vec{c}) = (6 - 3 - 1, 8 - 3 + \frac{5}{2}, 4 - 3 + \frac{1}{2})$$

$$= (2, 7\frac{1}{2}, 1\frac{1}{2})$$

$$e) |\vec{b}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$f) |\vec{c}| = \sqrt{2^2 + 5^2 + 1^2} = \sqrt{30}$$

2 (the "new" ~~one~~ on the handout)

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos \theta$$

$$\theta = \arccos \left(\frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} \right)$$

2. Angles $p = (3, 2, -6)$ $q = (4, -3, 1)$

$$a) \theta = \arccos \left(\frac{12 - 6 - 6}{\sqrt{3^2 + 2^2 + 6^2} \sqrt{4^2 + 3^2 + 1}} \right) = \arccos(0)$$

$$= 90^\circ$$

$$b) (4, -2, 4) \text{ and } (3, -6, -2)$$

$$\theta = \cos^{-1} \left(\frac{16}{6 \cdot 7} \right)$$

$$3. \quad PQ = (4-3, 4-1, 2-1) = (1, 3, 1)$$

$$AB = (3-1, 8-2, 3-1) = (2, 6, 2)$$

Parallel means dot product \cos (angle between)

$$\text{is } 1 \rightarrow \frac{PQ \cdot AB}{\|PQ\| \|AB\|} = 1$$

$$\cos^{-1}(1) = 0$$

$$4. \quad a) (1, 1, -1) \times (2, -1, 3)$$

$$= (2, -5, -3) \quad \|\cdot\| = \sqrt{4+25+9} \approx 6.16$$

$$b) (3, 2, 1) \times (-1, 0, 4)$$

$$= (8, -13, 2) \quad \|\cdot\| = \sqrt{8^2+13^2+2^2} \\ \approx 15.39$$

5. Unit vector \perp to two other vectors is norm of cross product

$$(a) \quad \|(3, 2, 5) \times (0, 3, -1)\| = \|-17, 3, 9\| \approx 19.4$$

So vector is $(-17, 3, 9)/19.4$

$$(b) (1, 1, -1) \times (0, 1, 2) = (3, -2, 1)$$

$$\|\cdot\| \approx 1.73 = \sqrt{9+4+1} = \sqrt{14}$$

$$\frac{1}{\sqrt{14}} (3, -2, 1)$$

$$6. A = (1, 1, 0) \quad B = (5, 2, 0)$$

$$\vec{x} = \vec{A} + t(\vec{B} - \vec{A}) \quad \vec{x} = \vec{B} + t(\vec{A} - \vec{B})$$

$$\vec{x} = \vec{A} + t(\vec{B} - \vec{A})$$

i.e.,

$$x = 1 + t(5-1) = 1 + 4t$$

$$y = 1 + t(2-1) = 1 + t$$

$$z = 0 + t(0) = 0$$

$$P = x\left(\frac{1}{2}\right) = \left(1 + 2, 1\frac{1}{2}, 0\right) = \left(3, \frac{3}{2}, 0\right)$$

$$Q = x\left(\frac{3}{2}\right) = \left(1 + 6, 2\frac{1}{2}, 0\right) = \left(7, \frac{5}{2}, 0\right)$$

$$R = x\left(-\frac{1}{2}\right) = \left(1 - 2, \frac{1}{2}, 0\right) = \left(-1, \frac{1}{2}, 0\right)$$

$$7. \quad \vec{x} = \vec{B} + t(\vec{A} - \vec{B})$$

i.e.,

$$x = 5 + t(1-5)$$

$$y = 2 + t(1-2) \quad \text{etc.}$$

$$z = 0$$